Quantum Topology

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"Quantum Topology" deals with the general quantum theory as the theory of quantum space. On the quantum level space time and energy momentum forms form a connected manifold; a functional quantum space. Many problems in quantum theory and field theory flow from not perceiving this symmetry and the functional nature of the quantum space.
Both topology, groups and logic are based on the concept of sets. If properties coincide with the open sets of topology, then logic and topology will have the same structure. If transformations are continuous in topology, then we will have topological groups; we can derive fields. Therefore, quantum logic underlies the manifold and the fields and nature is based on the language of quantum logic.
Quantum theory and field theory based on sets and the derived topology, group and logic structures should address the question of computation and the mind; the quantum computer and the quantum mind. 11.10.-z 11.15.-q 04.60.-m

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I. INTRODUCTION

Minkowski said: "*Henceforth space by itself and time by itself are doomed to fade away into mere shadows and only a kind of union of the two will preserve an independent reality*".
We can take a little step and see that the natural extension to that came with the quantum theory; that the space-time form and the conjugate energy-momentum form are not independent realities by themselves, they recede leaving behind a functional space; the quantum space; then topology dominates physics.
The role played by $\hbar$ in quantum theory, analogous to the role of C in relativity theory; it shows us more deeply the connectedness of the world. This view about the quantum is presented in the spirit of Minkowski's radical view of space and time.

A. The Quantum Question

There is a connection between quantum theory and relativity theory, take the relationship $\Delta E \Delta t \geq \hbar$, and suppose we have a physical system in the vacuum, its energy will go into change, it will look as if it went into Lorentzian transformation $LE \longmapsto E'$, its proper time will go into reciprocal transformation $L^{-1}T \longmapsto T'$, such that their product; the action integral itself will remain invariant. Let the system smoothly vanish $(E_0 \longmapsto 0)$, now in vacuum, fluctuations of energy will look as if it is accompanied by reciprocal fluctuations of time (as if a gravitational field fluctuate), such that $\hbar$ will be our invariant.
Energy-momentum fluctuations in vacuum will be accompanied by reciprocal fluctuations in the metric, the whole manifold will go into fluctuations, energy-momentum and space-time lose heir independence and become a connected manifold, a functional-quantum space. We are left for $\hbar$ alone; the invariant.
Why G ? If I may speculate about a ratio between $\hbar$ and cosmological action that makes G a weak coupling on a cosmological scale; just a trace of the direct coupling.

B. Optics Mechanics Analogy

$$\begin{aligned} k = \partial\varphi/\partial r, \qquad p = \partial S/\partial r, \qquad \hbar k = p, \\ \omega = \partial\varphi/\partial t, \qquad E = \partial S/\partial t, \qquad \hbar\omega = E. \end{aligned} \tag{1}$$

Purely geometric quantities (φ, k, ω) acquired dynamical meaning.

C. Waves, Particles and Probability

Waves that do not scatter, particles that do not localize, probabilities that interfere, the quantum sense of these concepts required a generalization beyond the classical meaning of these concepts to some underlying more basic concept.

D. General Relativity

Treating energy-momentum form symmetrically with the geometric aspect of nature in the functional space will lead us from an incomplete geometric view to a complete topological view.
Functional space seems to be the way out of the singularities of space-time: Functional have no value at separate points.
Quantization.

E. Vacuum State

Quantum fluctuations in the vacuum endowed the vacuum with a complete picture of the dynamics. Fluctuations in energy-momentum in vacuum means that the vacuum is not the simple geometric aspect of nature. Simple dimensional analysis shows so; it is $\hbar$ space; it is $\hbar$ that generates the dynamics. We can see that, the vacuum is the functional 'quantum' space. The pure quantum aspect of nature.

II. QUANTUM SPACE

T 1. Finiteness of $\hbar$:
(a) $q \times p \neq \emptyset$ (Means that both q and p are nonempty)
(b) If $q \times p = \emptyset$ Then $q = \emptyset$ Or $p = \emptyset$.
It is the finiteness of $\hbar$ which leads us to the functional space. In the classical limit $\hbar \longmapsto 0$, (2) is valid; p_μ , and q^μ , split into separate forms; so the classical action can not lead to a functional space.
A 1. *$\hbar$ is a finite measure that generates a functional space; the quantum space.*
Our starting point is the antilinear bilinear form (p_μ, q^μ) , a product of forms $\int p_\mu dq^\mu$, that build up the functional $\hbar$. It is obvious that this form has infinitely many states of representation. We can derive these states from the theory of representation.

III. REPRESENTATION SPACE

T 2. Reisz Representation Theorem:
Each linear functional ϕ on a Hilbert space H can be expressed:

$$\phi(\psi) = (\psi_\phi, \psi), \tag{2}$$

(ψ_ϕ, ψ) is an inner product and ψ_ϕ is an element of H^* uniquely determined by,

$$\psi_\phi = \int \phi(\psi_\phi)^* \psi_j dj. \tag{3}$$

A 2. *A product of a Hilbert space H with its dual space H^* in quantum theory $< AIA >$.*
So, the states and the Hilbert space of the quantum theory follow directly from the functional representation, with the difference that, only the functional $< AIA >$ is significant.

IV. PROJECTIONS

We can built up the space by taking the topological product $T_1 \times T_2$, of its subspaces.
T 3. Topological Theorem:
The transformations,

$$\hat{p} : T_1 \times T_2 \longmapsto T_1, \\ \hat{q} : T_1 \times T_2 \longmapsto T_2. \tag{4}$$

defined by,

$$\hat{p}(x_1, x_2) = X_1, \\ \hat{q}(x_1, x_2) = X_2. \tag{5}$$

are continuous, they are projections of the product space into its subspaces.
A 3. *In quantum space such mappings can be seen as quantum operators project the quantum space into one of its subspaces.* We start with Fourier integral relationship,

$$< p'IX >= \hbar^{-\frac{1}{2}} \int e^{-iq'p'/\hbar} dq' < q'IX >, \\ < q'IX >= \hbar^{-\frac{1}{2}} \int e^{iq'p'/\hbar} dp' < p'IX > . \tag{6}$$

to derive,

$$p_\mu = -i\hbar \partial / \partial q^\mu, \\ q^\mu = +i\hbar \partial / \partial p_\mu. \tag{7}$$

and see that; displacements:

$$- \partial / \partial q^\mu \longmapsto \hat{p}_\mu, \\ +\partial / \partial p_\mu \longmapsto \hat{q}^\mu. \tag{8}$$

are the corresponding quantum operators; projections of the quantum space.

$$i\hbar \hat{p}_\mu \longmapsto p_\mu, \\ i\hbar \hat{q}^\mu \longmapsto q^\mu. \tag{9}$$

V. FUNCTIONAL INTEGRALS

We can develop the representation in terms of Dirac δ-function; we take the set of basic vectors-forms and normalize,

$$< \xi' I \xi'' >= \delta(\xi' - \xi''). \tag{10}$$

the singular point $\xi' = \xi''$, should be seen as the condition that only the product of a basic vector-form with its conjugate, $\xi' = \xi''$, is permissible and finite (we may call it conjugate or functional product) only such product builds up a functional space, the δ-function will look like an infinite diagonal matrix, represents such functional.
We can see that the requirement of the product being functional normalized the space and removed certain type of infinity.
T 4. The δ-Space:
δ-function is a functional $(\delta, IA >)$ on the set of basic vectors-forms. It generates a functional space. Its representative

is the antilinear bilinear form $< AIA >$, we will call it the δ-space.
A 4. *Any quantum operator should be expanded in δ-space.* We have the basic equation,

$$(\delta, f) = \int f(x)\delta(x)dx = f(0). \tag{11}$$

for such an expansion, instead of the usual $< XIFIX >$ of the quantum theory. It is obvious that such an expansion in functional space will lead to distributions.
In the system of units $\hbar = \delta = 1$, $\hbar$-space is not different from δ-space. The states themselves transform to quantum operators and this is the physically significant space, the process of normalization carries on automatically to the operators themselves. The functional constraint imply that only conjugate interactions are permissible in the vacuum state; that means that no such thing as interaction between indeterminate number of particles.

VI. FEYNMAN FUNCTIONAL INTEGRAL

We have the basic equation,

$$(\delta, \varphi) = \int \varphi(q)\delta(q)dq = \varphi(0), \tag{12}$$

T 5. We can choose a convenient representation of δ-function, in terms of the action integral itself,

$$< q'Iq'' >= \delta(q' - q'') = e^{iS/\hbar}, \tag{13}$$

A 5. *This gives us*,

$$\int \varphi(q)e^{iS(q)/\hbar}dq = \varphi < O > . \tag{14}$$

This allow us to expand any function of a dynamical variable in δ-space in terms of the action integral.
(Analogous; Fourier Integral Theorem).

VII. EXTENDED SYMMETRY

Space-time are subject to Poincare,

$$x^\mu \longmapsto x'^\mu = a^\mu + l^\mu_\nu x^\nu. \tag{15}$$

Extension into the functional space imply other symmetries; the symmetries of the antilinear bilinear form (p_μ , q^μ) ; symmetry between p_μ and q^μ .
T 6. Topological Theorem:
The graph G , of a continuous transformation,

$$f : T_1 \longmapsto T_2. \tag{16}$$

is homomorphic with T_1 . The graph of the transformation is a subspace of the topological product $T_1 \times T_2$. We have the form (p_μ, q^μ) , p_μ's , transform as Dirac spinors $p_\mu = i\gamma^\mu\partial_\mu$,

$$\psi'(x) = exp(-\frac{i}{4}\sigma^{\mu\nu}\varepsilon_{\mu\nu})\psi(x). \tag{17}$$

transformations of the form (p_μ, q^μ) ,

$$p_\mu q^\mu \longmapsto p'_\mu q'^\mu. \tag{18}$$

A 6. *Will mix four Minkowski coordinates and four Dirac spinor displacements, the resulting group will be homomorphic with Poincare group.* People extensively explored such symmetry under the name "Supersymmetry".
C showed us the connectedness of space and time. $\hbar$ showed us the connectedness of p_μ and q^μ in the quantum space, it shows us the continuum that lies behind the discrete nature, reality itself is the quantum space, simple and beautiful.

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Quantum Topodynamics

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The basic set of the quantum space underlies the assumed oneness of topology, group and logic structure of quantum space. The elements of the basic set of the quantum space are identified as the Fourier components of dual linear forms D_λ , X^λ . Dynamics is identified as quantum mappings (commutation relations). The elements of the set -rays- are the basis of topology and the quantum computer, it encode the logic and perform functional integration, the physical picture based on holography permits for computability and the building up of the quantum computer. 11.10.-z 04.60.-m

I. THE SET

The infinite set of dual linear forms D_λ , X^λ , in which multiplication is defined in terms of topological, group and logical operation.

A. The Structure: First Order-Quantum Theory

The fundamental quantum conditions $[D_\lambda, X^\lambda] = -(\delta + i\omega)$, determines the structure of the functional space. In group theory, this is realized in graded Lie algebra, where, $p_\mu = -2(\gamma^\mu c)^{-1}\{Q_\alpha, Q_\beta\}$, $[Q^\alpha, Q_\beta] = -\delta^\alpha_\beta$. The commutation relations $[T^\alpha, T^\beta] = if^{\alpha\beta\gamma}T^\gamma$, determines the group structure of the 1-form D_λ .

B. The Structure: Second Order-Classical Theory

The commutation relations between 1-forms of the quantum-field determines the 2-form of the classical field, $T^\alpha.F^\alpha_{\lambda\nu} = ig^{-1}[D_\lambda, D_\nu]$.
Elements of the basic set are the Fourier components of D_λ , X^λ , they represent states; points or rays as $\psi_{\alpha o}$ in $H_{\alpha o}$. They are the basic building blocks of Logic, Topology and Group Structure.

II. THE LOGIC

A complete orthomodular lattice on the set, where subsets of L are the open sets of topology and transformations are continuous,

$$\begin{array}{ccc} & B - - - - - - -^{A} - - - - - - - L & \\ \sigma(g)| & & |s(g) \\ & B - - - - - - -^{A} - - - - - - - L & \end{array}$$

The 1-form D_λ encodes the logical structure α-properties, and dynamics $*$-morphisms, of fundamental laws and the fabrics of topology form a quantum-computer.
The Quantum Principle: *The infinite set of dual linear forms, that constitute the functional with their exponential information capacity is the basis of complexity.*

III. TOPOLOGY

The set is denumerable $(P < x|)P|y >=< x|y >$, and measurable $< A'|A'' >= (A' - A'')$.
In terms of the Fourier components, the functional space can integrate the dynamics; duality, and provide a global aspect of physics free of singularities and infinities that constitute the limit of using geometry and functions of analysis. The measure $\hbar$ determines the global property of the space, and the forms D_λ , ∂X^λ , are the local properties of the functional space. Both the variational principle and the gauge principle start from the functional; S , ω , to derive the local 2-form and, 1-form, respectively. The manifold and the fields are expressible in terms of this 1-form D_λ .

IV. THE FUNCTIONAL INTEGRAL

The basic idea here is functional δ-space. In mathematics δ is a linear functional on the set of test functions (δ, φ) , which may be interpreted as a linear functional on the set of basic vectors-forms $|A>$, or simply a functional δ-space, $< A|A >$. The integral admits representation as Feynman Integral,

$$\int \varphi(q) exp(iS(q)/\hbar) dq = \varphi < 0 >, \tag{1}$$

and the Fourier representation,

$$\frac{1}{2}\pi \int exp(ikx) dk, \tag{2}$$

and the group structure,

$$exp(igT^{\alpha}.\omega^{\alpha}(x)), \tag{3}$$

combined we get what we may call a holographic representation,

$$\int \varphi(q) exp(\int D_{\lambda} dq^{\lambda}) dq = \varphi < 0 > . \tag{4}$$

In terms of the Fourier components of D_{λ} . The continuous infinity of solutions depending on this form; or $\omega^{\alpha}(x)$, the choice of g's , each represents a quantum-holograph; an image of a universe existing in functional space. Topodynamics thus means quantum-holography. The exponential $\{igT^{\alpha}.\omega^{\alpha}(x)\}$, is factored into properties $\omega^{\alpha}(x)$, and bits g's . We may call this a quantum-hologram. Although there is an infinite Fourier components of the 1-form D_{λ} , in functional space there arise an infinite components of its dual ∂X^{λ} , and integration in terms of these twofold infinite components is computable and has topological meaning $(\delta, \omega, \Phi, T)$; functional, phase, flux, loop integral. The integral of the 1-form D_{λ} , is a different form; functional. The integral is not affected by the infinite components of D_{λ} , because of its dual X^{λ} , but is determined by the functional; bound by the topology.

Note: The important topological structure is encoded in $exp[\int D_{\lambda} \partial X^{\lambda}]$, the group and logical structure in $D_{\lambda} = \partial_{\lambda} - igW^{\alpha}_{\lambda}$.
Holography,
Solitons,
DNA,
SQUIDS,
Are candidates for the quantum computer.

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Differential Topology in Quantum Space

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To sum up, continuous transformations; Poincare-Lorentz, gauge, renormalization group, are needed to supplement functions and their function spaces, in fact it needed a continuous infinity of these and, their duals. The concept of functional δ-space -as a model- is a central idea to understand the transformations and the physical nature of the space. Functions and geometry are no longer representing physical reality.
(The curves ˜, and the waves — , are absent in the diagram, which represents the functional space.) Each point is in fact a ray, a Fourier component of D_λ , the metric itself should be represented in this way. This is a requirement of quantum mechanics, classical and quantum field theories and the Fourier representation of the space -an outcome of the transformations. The holographic principle recovers the Riemannian Einsteinian structure of the manifold. Charges act as solitons or holograms in the functional space; project a specific class of rays from the functional space through a certain phase angle $g\omega^\alpha(x)$, and here again the holographic principle recovers the geometric structure of the fields. 11.10.-z 04.60.-m

At each different neighboring point of the manifold there exists a locally inertial system of coordinates;

$$
\begin{array}{ccccccccccc}
 & & & & k^{(4)} & k^{(5)} & k^{(6)} & & & & \\
 & & & k^{(3)} & | & | & | & k^{(7)} & & & \\
 & & k^{(2)} & | & & & & | & k^{(8)} & & \\
 & k^{(1)} & | & & & \eta^{\mu\nu}_{(5)} & & & | & k^{(9)} & \\
 & | & & & & & & & & | & \\
<\omega^\alpha| & & & & & \delta^\alpha_\beta & & & & & |e_\beta> \\
 & & & & & & & & & & \\
 & & & & & \eta^{(n)}_{\mu\nu} & & & & &
\end{array}
$$

Dual basis, $[Q^\alpha, Q_\beta]$, $[\omega^\alpha, e_\beta]$, $[\eta^{\mu\alpha}, \eta_{\mu\beta}]$, $[g^{\alpha\mu}, g_{\mu\beta}]$, of δ^α_β .

The sequence of points, (1) , (2) ,....., there exist a sequence of fields, $\psi^{(1)}$, $\psi^{(2)}$,......, a sequence of Minkowskian metrics $\eta^{(1)}$, $\eta^{(2)}$, ..., and the continuous transformations connects a continuous infinity of these, (1) , (2) , ..., local systems. There is not one system (say space-time), but one at each point, and translations (rot) take place between neighboring systems in functional space. The generators of the transformations, say, $[I + \partial_\lambda(\delta x^\lambda)]$, or, $[I + i\partial_\lambda \omega(\partial x^\lambda)]$, are the dual basis of functional δ-space, $< \partial x^\alpha, \partial/\partial x^\beta >= \delta^\alpha_\beta$. In the language of Quantum Mechanics, Field Theory and Fourier representation, the 1-forms ∂_λ , ∂x^λ are k-waves. There is not a flat space-time $\eta_{\mu\nu}$ at each neighboring point of the manifold but a k-component of the basis forms ∂_λ , ∂x^λ . Quantum Mechanics and Field Theory require basically the language of the functional space and differential topology.

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Gauge Theory of Gravitation

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Gravitation as a quantum topological phenomenon on a large scale. Introducing the action of gravitation into functional space -through a phase angle- requires a change in our understanding of both gauge theory and general relativity. This displays its unity with the rest of gauge interactions and demonstrates that the manifold is compact. 04.60.-m 11.10.-z

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I. INTRODUCTION

The dimensionless coupling $G(x) = km/r$, plays the role of phase angle of cosmological origin; coupling between dipole moments. We can derive its action in functional space by introducing it into the group structure, $exp\{igT^{\alpha}.\omega^{\alpha}(x)\}$. We may use the notation $\{^{\alpha}_{\beta}\} = g^{\alpha\mu}g_{\mu\beta}$, and its contracted form $\{\}$, to represent this phase angles and to facilitate the calculations in terms of basis $\{\omega^{\alpha}(p)\}$, $\{e_{\beta}(p)\}$, and phase angles $< \omega^{\alpha}|e_{\beta} >$.
The phase angles exist in functional space and its contractions equal in the geometric language to its projections on the metric. Action of the phase angles reconstruct the wavefront and this determines the geometric structure; of the manifold and the fields.
We will now consider that these phase angles perform rotations that are subject to same algebraic scheme as the fields W . This sort of rotations will mix up fields in order to bring in the transformations on these fields. Furthermore the transformations will not change the structure of physical law. This means that the phase $\omega^{\alpha}(x)$, that is involved is a general one and $G(x)$, is associated with the same algebraic structure of T^{α}. This will permit these transformations to induce the appropriate transformations on the fields. Therefore we will assume,

$$exp\{iG(x)T^{\alpha}.\omega^{\alpha}_{\beta}(x)\}, \tag{1}$$

$$[T^{\alpha}, T^{\beta}] = if^{\alpha\beta\gamma}T^{\gamma}, \tag{2}$$

The infinitesimal transformations;

$$(I + iT^{\alpha}.\omega^{\alpha}_{\beta}(x) + iT^{\alpha}.\omega^{\alpha}(x) - iT^{\alpha}f^{\alpha\beta\gamma}\omega^{\alpha\beta}_{\beta}\omega^{\gamma}). \tag{3}$$

where,

$$\omega'^{\alpha}_{\beta} = G\omega^{\alpha}_{\beta}, \qquad \omega'^{\alpha} = g\omega^{\alpha}, \qquad W'_{\lambda} = gW_{\lambda}.$$

$$\underline{D}_{\lambda} = \partial_{\lambda} - iT^{\alpha}\Gamma^{\alpha}_{\lambda\beta} - iT^{\alpha}W^{\alpha}_{\lambda} - iT^{\alpha}W^{\alpha\alpha}_{\lambda\beta}. \tag{4}$$

$$\underline{W}^{\alpha}_{\lambda} = (\Gamma^{\alpha}_{\lambda\beta} + W^{\alpha}_{\lambda} + W^{\alpha\alpha}_{\lambda\beta}). \tag{5}$$

$$\underline{\omega}^{\alpha} = \omega^{\alpha}_{\beta} + \omega^{\alpha} - f^{\alpha\beta\gamma}\omega^{\alpha\beta}_{\beta}\omega^{\gamma}. \tag{6}$$

where,

$$\Gamma^{\alpha}_{\lambda\beta} = \partial_{\lambda}\omega^{\alpha}_{\beta}, \qquad W^{\alpha\alpha}_{\lambda\beta} = f^{\alpha\beta\gamma}\omega^{\alpha\beta}_{\beta}W^{\gamma}_{\lambda}.$$

These expressions generalize D_{λ} , W^{α}_{λ} , ω^{α} , in the standard Yang-Mills formalism,

$$\underline{W}^{\alpha}_{\lambda} \longmapsto \underline{W}^{\alpha}_{\lambda} + \partial_{\lambda}\underline{\omega}^{\alpha} - f^{\alpha\beta\gamma}\underline{\omega}^{\beta}\underline{W}^{\gamma}_{\lambda}. \tag{7}$$

$$\psi^*(i\gamma\underline{D} - m)\psi. \tag{8}$$

Points:
$\partial_\lambda\underline{\omega}^\alpha$: Topological nature of the source.
$f^{\alpha\beta\gamma}\underline{\omega}^\beta\underline{W}^\gamma_\lambda$: Topological nature of the transformations,performed by the functional $\underline{\omega}^\alpha$, and, isovectors and Riemannian manifold.

$$W^{\alpha\alpha}_{\lambda\beta} = \frac{1}{2}g^{\alpha\mu}(W^\alpha_\lambda g_{\mu\beta} + W^\alpha_\beta g_{\mu\lambda} - W^\alpha_\mu g_{\lambda\beta}). \tag{9}$$

$$f^{\alpha\beta\gamma}\partial_\lambda\omega^{\alpha\beta}_\beta\omega^\gamma = \Gamma^\alpha_{\lambda\beta}\omega^\alpha, \tag{10}$$

$$g^{\alpha\mu}\partial_\lambda g_{\mu\beta}\omega^\alpha = g^{\alpha\mu}\partial_\lambda g_{\mu\beta}g^{\mu\beta,\alpha}g^\alpha_{\mu\beta}. \tag{11}$$

II. YANG MILLS EINSTEIN GAUGE FIELD

From the commutator $[\underline{D}_\lambda, \underline{D}_\nu]$,

$$\begin{array}{lllllll} F^\alpha_{\lambda\mu} & +\partial_\lambda & W^\alpha_\nu & -\partial_\nu & W^\alpha_\lambda & +f^{\alpha\beta\gamma} & W^\beta_\lambda & W^\gamma_\nu \\ & +\partial_\lambda & W^{\alpha\gamma}_{\nu\delta} & -\Gamma^\gamma_{\nu\delta} & W^\alpha_\lambda & +f^{\alpha\beta\gamma} & W^\beta_\lambda & W^{\gamma\gamma}_{\nu\delta} \\ & +\Gamma^\alpha_{\lambda\beta} & W^\alpha_\nu & -\partial_\nu & W^{\alpha\alpha}_{\lambda\beta} & +f^{\alpha\beta\gamma} & W^{\beta\alpha}_{\lambda\beta} & W^\gamma_\nu \\ & +\Gamma^\alpha_{\lambda\beta} & W^{\alpha\gamma}_{\nu\delta} & -\Gamma^\gamma_{\nu\delta} & W^{\alpha\alpha}_{\lambda\beta} & +f^{\alpha\beta\gamma} & W^{\beta\alpha}_{\lambda\beta} & W^{\gamma\gamma}_{\nu\delta} \\ & +\partial_\lambda & \Gamma^\gamma_{\nu\delta} & -\partial_\nu & \Gamma^\alpha_{\lambda\beta} & +f^{\alpha\beta\gamma} & \Gamma^\alpha_{\lambda\beta} & \Gamma^\gamma_{\nu\delta}. \end{array} \tag{12}$$

Conformity of this structure with Yang-Mills is brought about by complexity of the transformations that G^α_β performs, in conformity with structure of the space itself.

$$F^\alpha_{\lambda\nu} \longmapsto F^\alpha_{\lambda\nu} - f^{\alpha\beta\gamma}\omega^\beta F^\gamma_{\lambda\nu}. \tag{13}$$

The linear terms in lines 3 and 4,

$$\begin{array}{l} +\partial_\lambda g^\alpha_\beta W^\alpha_\nu + \partial_\beta g^\alpha_\lambda W^\alpha_\nu + \partial_\nu W^{\alpha\alpha}g_{\lambda\beta} \\ -\partial^\alpha g_{\lambda\beta}W^\alpha_\nu - \partial_\nu W^\alpha_\beta g^\alpha_\lambda - \partial_\nu W^\alpha_\lambda g^\alpha_\beta. \end{array} \tag{14}$$

$$\nu\bot\left(\lambda, \beta, \alpha\right).$$

$$\begin{array}{l} +\partial_\lambda g^\alpha_\beta + \partial_\beta g^\alpha_\lambda - \partial^\alpha g_{\lambda\beta}(W^\alpha_\nu g^\gamma_\delta + W^\alpha_\delta g^\gamma_\nu - W^{\alpha\gamma}g_{\nu\delta}) \\ -\partial_\nu g^\gamma_\delta + \partial_\delta g^\gamma_\nu - \partial^\gamma g_{\nu\delta}(W^\alpha_\lambda g^\alpha_\beta + W^\alpha_\beta g^\alpha_\lambda - W^{\alpha\alpha}g_{\lambda\beta}). \end{array} \tag{15}$$

$$(\nu, \delta, \gamma)\bot\left(\lambda, \beta, \alpha\right).$$

Lines 2, 3, and 4, show some new properties of $\{^\alpha_\beta\}$, as they mediate the way between the linear and the bilinear terms in $F^\alpha_{\lambda\nu}$. This can be interpreted as $\underline{\omega}^\alpha$ wave equation as distinct from $\underline{W}^\alpha_\lambda$ wave equation.

Note: Further work on,

$$\underline{D}_\lambda^{\alpha\beta} F_{\lambda\nu}^\beta = g \underline{J}_\lambda^\alpha. \tag{16}$$

$$[\underline{D}_\mu, [\underline{D}_\lambda, \underline{D}_\nu]] + [\underline{D}_\nu, [\underline{D}_\mu, \underline{D}_\lambda]] + [\underline{D}_\lambda, [\underline{D}_\nu, \underline{D}_\mu]] = 0. \tag{17}$$

$$\partial_\lambda F_{\lambda\nu}^\alpha = f^{\alpha\beta\gamma} \underline{W}_\lambda^\gamma F_{\lambda\nu}^\beta. \tag{18}$$

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